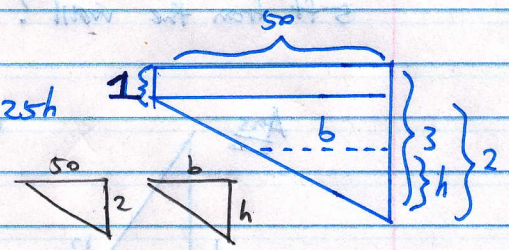
10/25/12

Eg) A swimming pool is 50m long and 20m wide. It's depth decreases along the length from 3m to 1m. If it initially empty and filled out at a rate of $1m^3/min$. How fast is the water level rising 250min after the filling begins?

How long will it take to fill the Pool?



Ans By similarity of $\triangle$'s $\frac{2}{50} = \frac{h}{b} \Rightarrow b = 25h$

$A = \frac{1}{2}bh = \frac{1}{2} \cdot 25h \times h = \frac{25}{2}h^2$

Volume $= V(h) = 12.5h^2 \times 20 = 250h^2$

$A \times w$ (w: width)

After 250 mins, volume $= 250m^3 = 250h^2 \Rightarrow h^2 = \frac{250}{250} = 1$

$\Rightarrow h = 1$ (At $t = 250$min, $h = 1$)

$1 \Rightarrow \frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = 500h \cdot \frac{dh}{dt} = 500 \cdot 1 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{500} = 0.002m/min$

(water is filled at the rate of $1m^3/min$)

$\frac{dh}{dt} = 2mm/min$

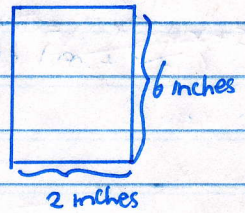
$\therefore$ After 250min, the water is rising at the rate of 2mm/min //

Time taken to fill the pool : use volume of the pool + rate of filling $= 1m^3/min$ //

(volume $= 50 \times 20 \times 2 = 2000m^3$) ($1m^3/min$)

Eg) At what rate is soda being sucked out of a cylindrical glass that is 6 inches tall and has a radius of 2 inches? The depth of the decreases at a constant rate of 0.25 inches/sec.

Ans



Should find $\frac{dV}{dt}$, V = volume

Given $\frac{dh}{dt}$, h = height of soda in the glass.

Volume $= \pi r^2 h$, $r = 2in$

$\frac{dh}{dt} = -0.25$ in/sec

$V = 4\pi h$; $\frac{dV}{dt} = 4\pi \cdot \frac{dh}{dt}$

$= 4\pi(-0.25)$

$= -\pi$ in^3/sec

$\therefore$ Hence Soda is being sucked out at the rate of π inch^3/sec //